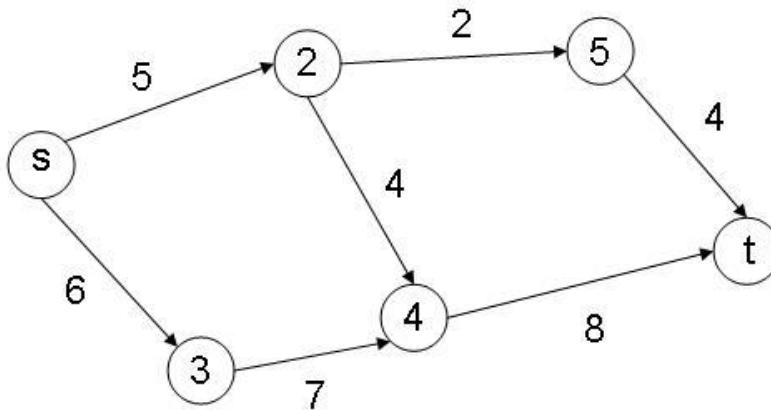


No collaboration with others, nor use of material other than the textbook and class notes, are allowed. Enjoy!

- For the following graph, find the maximum flow and minimum cut by applying Ford-Fulkerson by hand. Show the intermediate graphs and residual graphs.



- Suppose you are given a flow network where each edge has the same capacity, that is, a directed graph $G = (V, E)$ with a source node s and sink node t , such that $c_e = j$ for all $e \in E$, where j is an integer constant greater than zero. You can reduce the maximum s - t flow by removing edges.

Given $k < |E|$, find the k edges whose removal from the graph reduces the maximum s - t flow the most. That is, describe the algorithm that takes a flow network $G = (V, E)$ and a number k , and returns the set of edges E' such that $E' \subset E$ and $|E'| = k$, and the maximum s - t flow in $(V, E - E')$ is as small as possible.

Your algorithm should be polynomial in $|V| + |E|$, the number of vertices and edges in the graph. You can assume you have the Ford-Fulkerson algorithm to build on.

- Let $G = (V, E)$ be a flow network with source s , sink t and integer capacities. Suppose that we are given a maximum flow in G .
 - Suppose that the capacity of a single edge $(u, v) \in E$ is increased by 1. Give an $O(|V| + |E|)$ time algorithm to update the maximum flow.
 - Suppose that the capacity of a single edge $(u, v) \in E$ is decreased by 1. Give an $O(|V| + |E|)$ time algorithm to update the maximum flow.