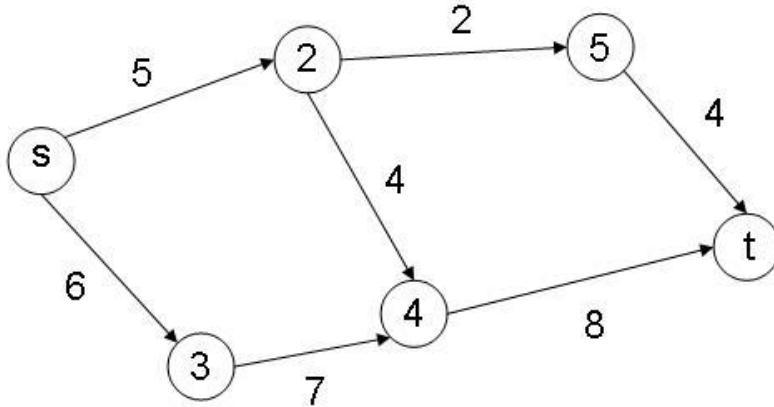


No collaboration with others, nor use of material other than the textbook and class notes, are allowed. Enjoy!

1. For the following graph, find the maximum flow and minimum cut by applying Ford-Fulkerson by hand. Show the intermediate graphs and residual graphs.



2. Suppose you are given a flow network where each edge has the same capacity, that is, a directed graph $G = (V, E)$ with a source node s and sink node t , such that $c_e = j$ for all $e \in E$, where j is an integer constant greater than zero. You can reduce the maximum s - t flow by removing edges.

Given $k < |E|$, find the k edges whose removal from the graph reduces the maximum s - t flow the most. That is, describe the algorithm that takes a flow network $G = (V, E)$ and a number k , and returns the set of edges E' such that $E' \subset E$ and $|E'| = k$, and the maximum s - t flow in $(V, E - E')$ is as small as possible.

Your algorithm should be polynomial in $|V| + |E|$, the number of vertices and edges in the graph. You can assume you have the Ford-Fulkerson algorithm to build on.

3. Let $G = (V, E)$ be a flow network with source s , sink t and integer capacities. Suppose that we are given a maximum flow in G .
 - (a) Suppose that the capacity of a single edge $(u, v) \in E$ is increased by 1. Give an $O(|V| + |E|)$ time algorithm to update the maximum flow.
 - (b) Suppose that the capacity of a single edge $(u, v) \in E$ is decreased by 1. Give an $O(|V| + |E|)$ time algorithm to update the maximum flow.